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Yupana

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Yupana is the name given to the counting device – or abacus – of the Incas; in the indigenous Andean language of Quechua, "yupay" means "to count." Counting was a fundamental aspect of structuring such an extensive and elaborate civilization as that of the Incas, which reached its height in the fifteenth and sixteenth centuries with an estimated population in the millions and extending across the diverse climatic regions of northern Chile and Argentina, Ecuador, Bolivia, Peru, and the southern part of Colombia. This calculating device along with its corresponding recording device – the quipu – were the main technologies that organized the Inca population and controlled the distribution of goods and labor responsibilities. While the quipu knot records were static registries of amounts calculated and served as the authoritative voice of quantities paid, quantities due, and standard measurements and statistics, the yupana would have been the device used to calculate (but not register) those amounts later tied into quipu cords. While the knot structure of the quipu was not suitable for the manipulation of numbers nor performing repeated algorithms, the yupana is a table upon which counters (of beans, corn, pebbles, grains, or other tokens) can be freely moved, thus facilitating the execution of distinct numeric operations.

Colonial Descriptions of La Yupana

The yupana has been associated with two different artifacts: (1) the carved rock, wooden, and clay deposits of archaeological provenience and the (2) sketched figures of counting boards primarily from Guaman Poma de Ayala's *Nueva corónica y buen gobierno*, both of which are illustrated and discussed below. The interpretation of these Inca artifacts as counting tools is supported by ample documentation of the existence of such a device in numerous colonial chronicles and texts. The Spanish missionary José de Acosta describes (without understanding) the Inca counting system:

> To see them use another kind of quipu with maize kernels is a perfect joy. In order to carry out a very difficult computation for which an able accountant would require pen and ink. . . these Indians make use of their kernels. They place one here, three there and eight I do not know where. They move one kernel here and three there and sure enough, they are able to complete their computation quickly without making the smallest mistake. As a matter of fact, they are better at calculating what each one is to pay or give than we would know how to check with pen and ink. Whether this is not ingenious and whether these people are wild animals let those judge who will! What I consider as certain is that in what they undertake they have great advantage over us. (De Acosta, 2008, p. 211)

Archaeological Artifacts with Carved Deposits for Counting

The first archaeological finding linked to the yupana was a symmetric tablet (33×27 cm) found near Chordeleg (Cuenca) in 1869 exhibiting 21 carved spaces and elevated platforms on two diagonal corners (Fig. 1). Similar objects have been uncovered in the ruins of Chan Chan, between Cuenca

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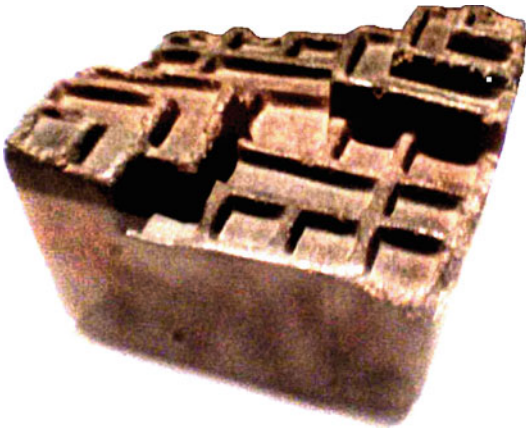

Fig. 1 A carved deposit similar to the Chordeleg yupana

and Sig Sig, Ancash, Caraz, Pallasca, Callejón de Huaylas, and Ica (Radicati di Primeglio, 1990, pp. 220–229). Although these objects have been classified as counting devices by certain scholars (Arriaga, 1922; Gonzales Suarez, 1878; Verneau and Rivet, 1912; Wiener, 1880), this is only one possible interpretation. At one point these carved deposits were also believed to be architectonic models (Bastian (n.d.); Gonzales Suarez, 1878) as well as games of chance (Uhle, 1922).

La Yupana de Guamán Poma de Ayala

The yupana illustrated by Guamán Poma de Ayala, however, has unanimously been regarded as a counting device (whether or not scholars agree to Guamán Poma's knowledge of its functionality). Hugo Pereyra Sánchez attributes this mutual understanding to the context in which the abacus is illustrated: "In the Guamán Poma illustration the board, without a doubt, accompanies and complements the quipu that the accountant and treasurer extends. In light of such precise testimony, it would be arbitrary to place in doubt the fact that the object represented is an instrument of calculation with which operations were made and whose results were recorded on the quipu" (Radicati di Primeglio, 1990, p. 235 my translation). Henry Wassén concurs by denying that the yupana "deals with a game board... there is nothing in common between Guaman Poma's drawing and the game boards reproduced by Verneau and Rivet" (Wassén, 1990, p. 214 my translation). This yupana is a 5 × 4 table etched into the ground or similar flat surface upon which tokens are moved (Guaman Poma specifically indicates that the grain quinoa is used). The columns have distinct numbers of empty (white) and filled (black) spaces (5, 3, 2, and 1), although various scholars, including Pereyra, have proposed that the black tokens are indicative of addition/debits and the white tokens are indicative of subtraction/credits (Fig. 2).

The distinct positional values given to each row and column of the yupana have led to diverse models and interpretations of this abacus. Wassen's initial interpretation (1931) (Fig. 3) has subsequently been disregarded as overly complicated and not one to one (i.e., numbers could be represented by multiple token combinations). Pereyra's model (1990) associates column position value with the etchings on the board (5, 3, 2, and 1) while preserving the decimal progression of the rows (units, tens, hundreds, thousands, ten thousands) (Fig. 4). While this is one way to make sense of the inscribed 5-3-2-1 sequence, this abacus would only require the placement of 1–2 tokens in each box which does not seem congruent with the specific number of empty spaces illustrated by Guamán Poma. Radicati di Primeglio's model (1979) gives each column the same multiplicative

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Fig 2 Guaman Poma de Ayala's illustration of the yupana (*bottom left corner*)

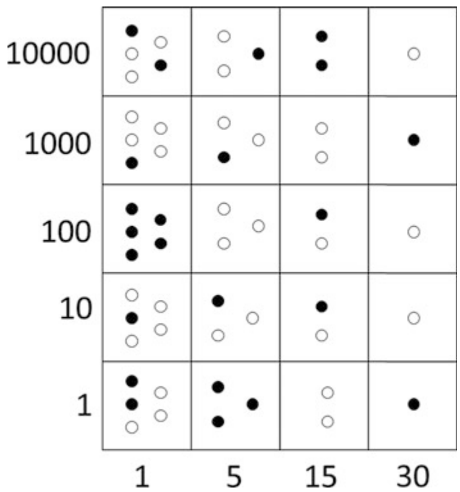

Fig. 3 Wassen's solution (1931). According to this model, Guamán Poma's yupana would represent the value: 408,257 = 2 × 1 + 3 × 5 + 1 × 30 + 1 × 10 + 1 × 50 + 1 × 150 + 5 × 100 + 1 × 1,500 + 1 × 1,000 + 1 × 5,000 + 1 × 30,000 + 2 × 10,000 + 1 × 50,000 + 2 × 150,000 (Image by author)

positional value (x1) and mirrors the way in which quipu cords represented numeric values (Fig. 5); this model would allow for the simultaneous placement of four numbers (one in each column) but would also require surpassing the limit of token positions as illustrated (5, 3, 2, and 1). The Burns model (1981) preserves the same positional values but only represents one number at a time, thus

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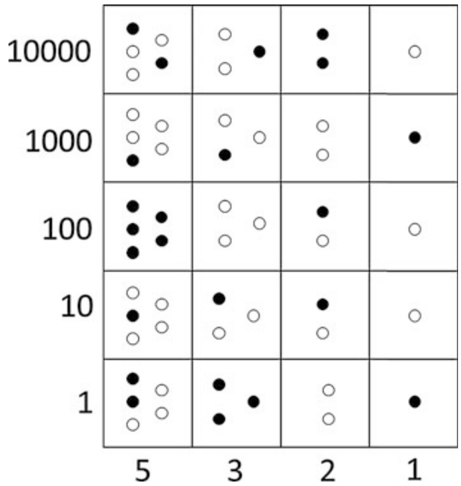

Fig. 4 Pereyra's proposal (1990). According to this model, Guamán Poma's yupana would represent the value: 181,826 = 2 × 5 + 3 × 5 + 1 × 1 + 1 × 50 + 1 × 30 + 1 × 20 + 5 × 500 + 1 × 200 + 1 × 5,000 + 1 × 3,000 + 1 × 1,000 + 2 × 50,000 + 1 × 30,000 + 2 × 20,000 (Image by author)

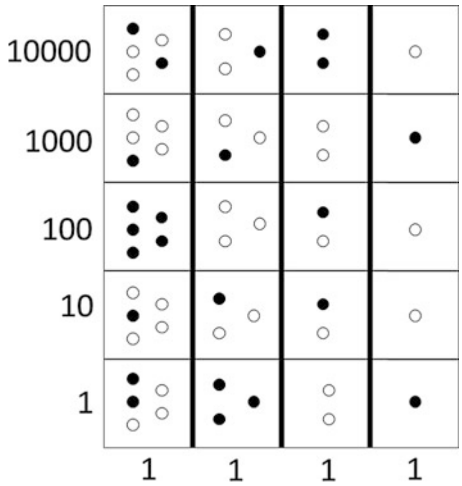

Fig. 5 Radicati di Primeglio's version (1979). According to this model, Guamán Poma's yupana would represent four values: 21,512, 11,013, 20,110, and 1,001 (Image by author)

effectively making use of positions 5, 3, and 2, with the 1-hole space an extra "memory" space intended for carrying over vales between the units, tens, hundreds, thousands, and ten thousands positions (Fig. 6). Burns also somewhat arbitrarily rotated the abacus on its side, but this position's correlation with the contemporary practice of writing numbers from left to right (along with the abacus's use of the decimal system) makes this model a popular choice for implementation in elementary schools (Fig. 7).

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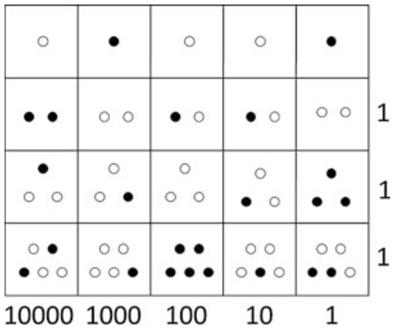

Fig. 6 Burn's method. According to this model, Guamán Poma's yupana would represent the value: 53,636 or 62,645 (if the *top row* "memory holes" carried a token of higher place value); the placement of tokens in the top row ultimately indicates that the operation is not yet finalized (Image by author)

Fig. 7 First-grade students from the *Institución Educativa 3043 Ramón Castilla* using the yupana in the classroom under the direction of their instructor Miguel Angel Díaz Sotelo, the coordinator (Peru) of the Latin-American Network of Ethnomathematics (see http://www.etnomatematica.org/home/) (Photos reproduced with his kind permission)

Performing Operations on the Yupana

The model proposed by the author (Leonard 2010) is a variation of the Burns model and essentially operates in the same way but positions the yupana as illustrated by Guamán Poma in order to preserve correspondence between the yupana and the quipu in which numbers are read from top to bottom (Figs. 8, 9, 10 and 11).

Addition (as executed in Fig. 7) is the most straightforward of all calculations and probably one of the most common operations. One of the summands is placed on la yupana while the other summand is placed outside la yupana, to the right of the corresponding rows (A). The stones outside the abacus in the ones row are added to la yupana first (B), using the memory stone as needed. Whenever the total number of stones in any given row reaches ten (hence, the 5 square, 3 square, and 2 square are all full), then those stones can be removed and one stone placed in the single hole (the memory hole) on the right (C). The rest of the stones can then be added to the now-empty holes of that same row (D). After all the stones have been added, the stone from the memory hole can be shifted up to occupy a space in the next row up (E), and the process continues by adding the summand from that

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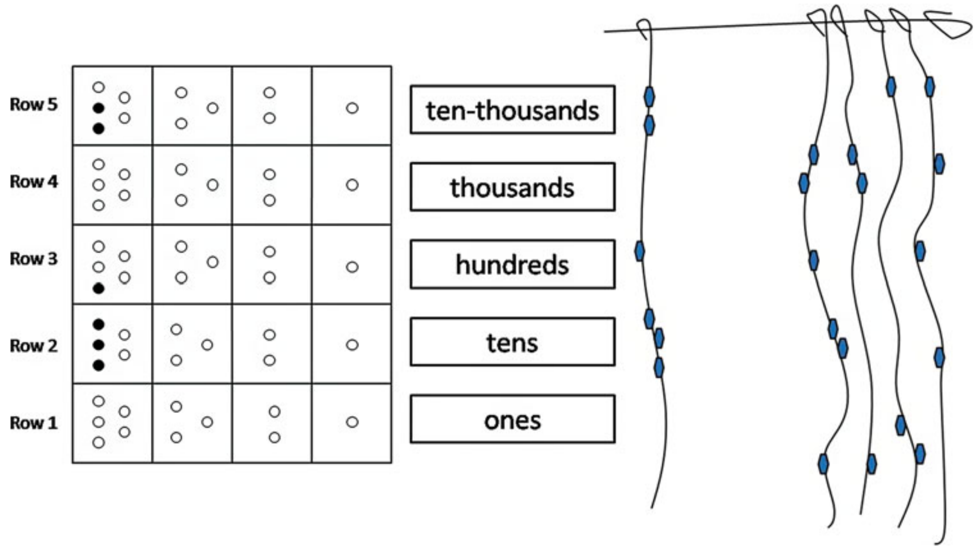

Fig. 8 Showing how the abacus, when positioned vertically, corresponds directly with quipus cord values. This image shows how the number 20130 is displayed on la yupana as well as the first cord of the quipus (Image by author, published in Leonard 2010, p. 92)

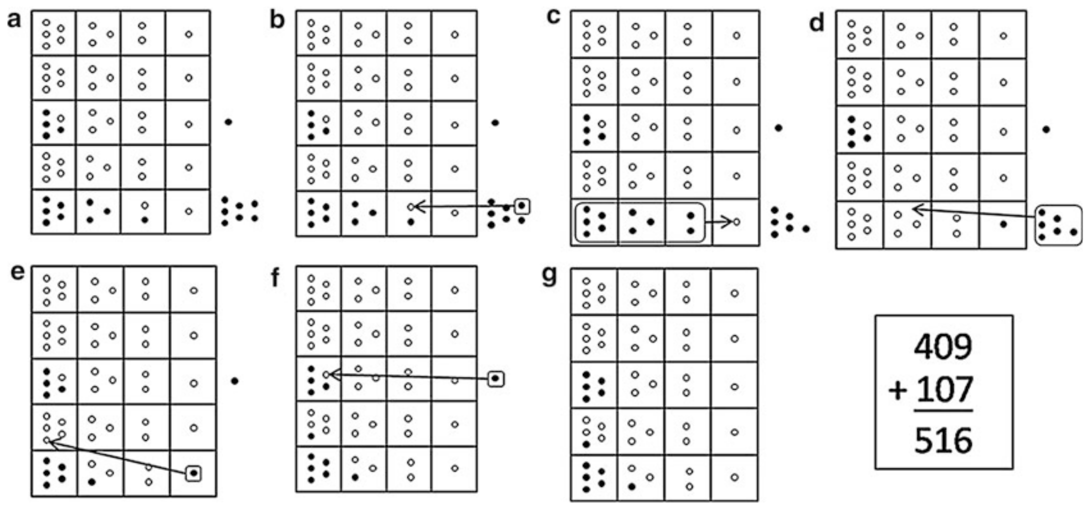

Fig 9 Addition on the yupana (Image by author, published in Leonard 2010, p. 95)

next row. All the place values are computed in the same fashion, always moving from the bottom to top. Once all the summand stones on the right side of the abacus have been worked into the calculation, the number remaining on la yupana is the answer, in our case 516.

Subtraction is a similar computation, but the steps are done in reverse. Multiplication and division are also possible using this method but require the use of additional yupana to prepare some partial products needed in the operation. These operations can be viewed in their entirety in this author's article "The Incan Abacus" on the website of the *Journal of Mathematics and Culture* (volume 5, number 2): http://nasgem.rpi.edu/pl/journal-mathematics-culture-volume-5-number-2. Here, an interesting connection between the enigmatic sequence 5-3-2-1, Fibonacci's sequence, partial products, number decomposition, and Aymara linguistics is also established.

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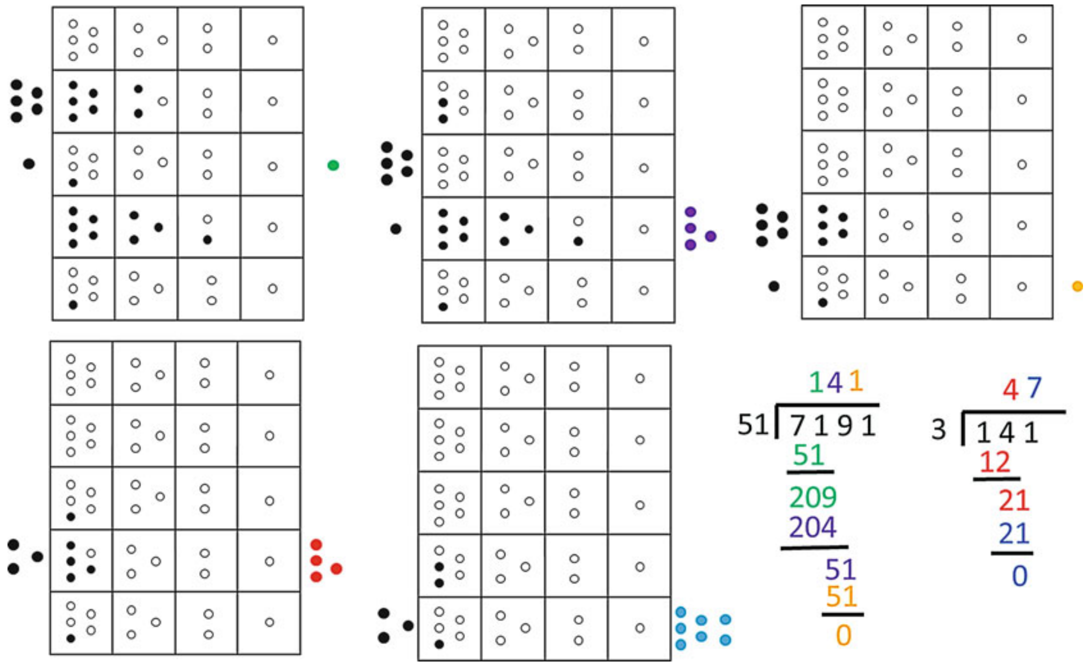

Fig. 10 Division simplified by incorporating mental mathematics and factorization (i.e., 7,191/153 = 7,191/(51 × 3) = (7,191/51)/3). The long division illustrated alongside the yupana is intended to serve as a comparison, even though pen and paper are not necessary for performing this operation (Image by author)

Final Reflections and Operation Simplification

Further reflection on the functionality of the yupana, however, questions the need to establish the possibility of performing addition, subtraction, multiplication, and division. Most models have been criticized for the complexity of the multiplication and division operations, but until additional evidence of the exact nature of Inca operations and necessities of calculation is established, it is difficult to know which exact operations would have been carried out on the abacus. In addition, we must accept that those using the abacus would have been highly trained operators, greatly reducing what constitutes complexity to them. Finally, if we consider the possibility of the use of strategic mental mathematics and factorization, these abacus operations can be greatly reduced and as simple as those calculated using pen and paper, as seen by comparing a simplified version of division (Fig. 10) with a Burn's inspired model of division (Fig. 11).

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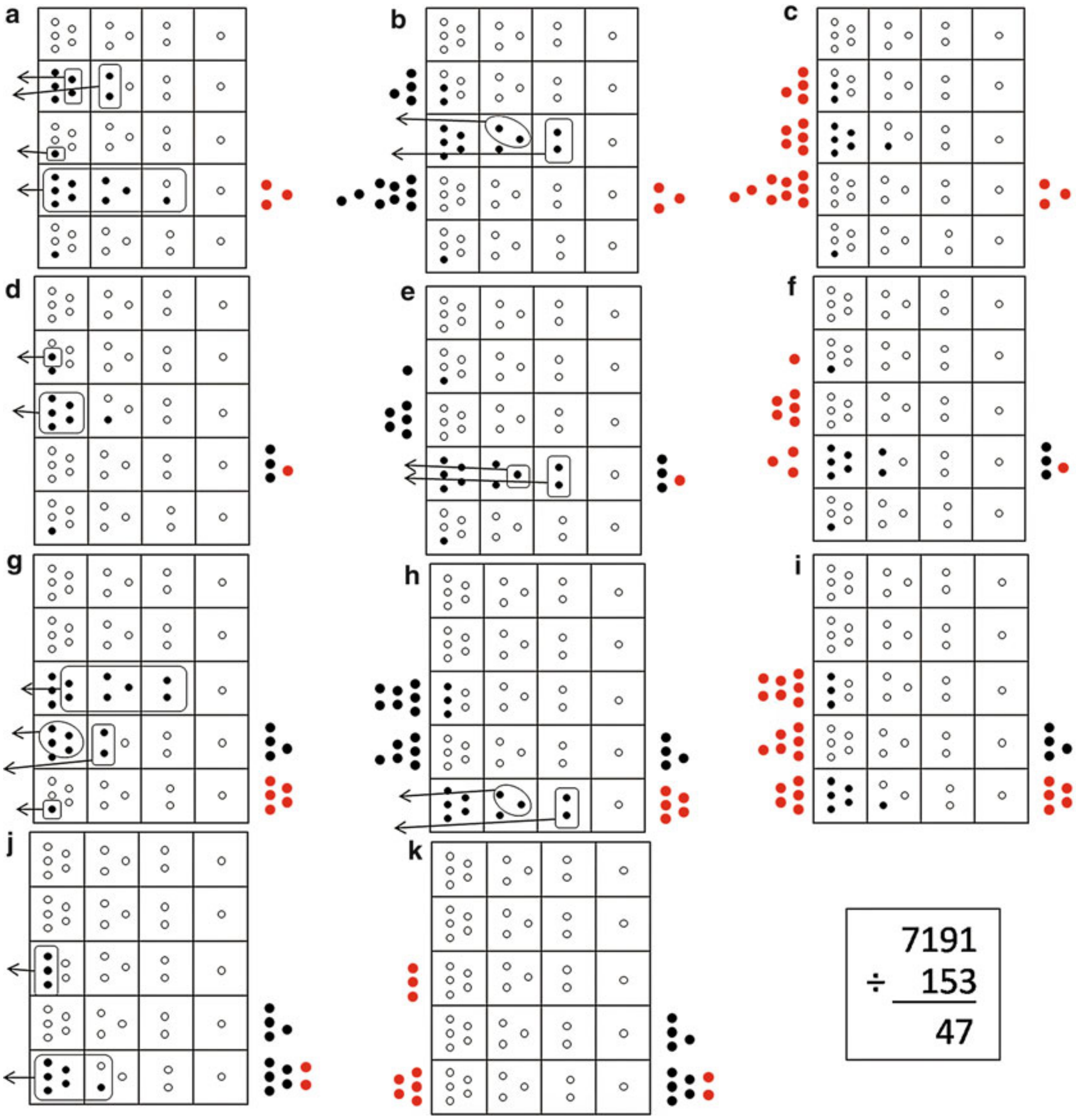

Fig. 11 Division on the yupana (Image by author, published in Leonard 2010, p. 102)

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